

## **Games for Enhancing Sustainability of Year 7 Maths Classes in Indonesia Theory-Driven Development, Testing and Analyses of Lessons, and of Students' Outcomes**

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### ***Abstract***

The results of international comparative studies have shown that relationships exist between metacognition and cognitive activation and learning success. Since 2007 we have been carrying out projects in Indonesia to improve cognitive and metacognitive activities of pupils of year 7 and their teachers. These activities are to contribute to the construction and sensible use of sustainable mental models for mathematical concepts and methods by learners. This paper shows how games are used for the enhancement of metacognitive and discursive activities in class. Their effectiveness is documented exemplarily by means of students' outcomes and transcripts of lessons from project classes.

**Keywords:** Cognitive activation, Metacognition, Games, Integers

### ***Abstrak***

Hasil dari studi banding internasional telah menunjukkan bahwa di satu sisi terdapat hubungan antara metakognisi dan kegiatan kognisi dan pada sisi lain terdapat keberhasilan belajar. Sejak tahun 2007 telah dilakukan projek di Indonesia untuk meningkatkan kognisi dan kegiatan kognisi dari siswa kelas 7 beserta para pengajarnya. Kegiatan ini harus memberikan kontribusi untuk pembangunan dan penggunaan yang dapat dimengerti dari sebagian besar bentuk ide tentang pengertian secara matematik dan metode-metode melalui siswa. Pada artikel ini digambarkan bagaimana permainan diterapkan sebagai pendukung metakognitif dan kegiatan diskursif dalam pengajaran. Pengaruhnya didokumentasikan berdasarkan contoh dari hasil kerja siswa dan catatan pengajaran dari kelas proyek.

**Kata kunci:** Pengaktifan kognitif, Metakognisi, Permainan, Bilangan bulat

### ***Introduction***

For more than a decade, Indonesian mathematics educationalists endeavour with a variety of different methods to improve mathematical abilities of Indonesian students. The most important project, which is broadly conceived in primary schools (Year 1 to 6) in this reform process, is called PMRI (Sembiring et al., 2010). In 2009, the Institute of Cognitive Mathematics (Osnabrück University, Germany) assumed

responsibility for the German-Indonesian feasibility study “Development of Metacognitive and Discursive Activities in Indonesian Maths Teaching” (MeDIM) in Year 7 in cooperation with the Institut Mathesis (Pyzdry, Poland) and the Institute for Didactics in Mathematics (Sanata Dharma University, Yogyakarta). With the introduction of cognitive, metacognitive and discursive activities in this study, some methodological approaches approved from PMRI were further continued. The documentation of the theoretical foundations of the project MeDIM, the concept of new learning environments and exercises as well as the first results of this project can be found in Kaune et al. (2011, 2012).

For the introduction of integers, two learning environments have been constructed in MeDIM. These use a realistic context to create and organise mathematical knowledge in the students’ minds and to introduce the handling of mathematical theories. The first learning environment was created as a “bank” with the main focus of accounting of credit and debts on a bank account. The second learning environment was designed as an activity called “Jumping back and forth on the number line”. For further reading, similar approaches for these learning environments have been published (cf. Streefland, 1996; Kliemann et al., 2007).

To make a lasting improvement on students’ mathematical abilities, the existence of new materials and exercises in lessons alone is not sufficient (cf. Sembiring et al., 2008, p. 928). Furthermore, the behaviour of the learners has to be influenced, so that they are being educated to practice metacognitive and discursive activities (Kaune & Nowinska, 2011) in order to adopt and use the mental models for mathematical concepts offered. This explains why these learning environments contain materials and exercises that increasingly evoke students’ metacognitive skills and encourage cooperative, discursive forms of work (Kaune et al., 2011).

The feasibility study MeDIM showed that Indonesian students are able to perform better which previously has been considered impossible by mathematics educationalists and teachers. The afore mentioned learning environments for the development of sustainable mental models for mathematical concepts and methods, and the growing endeavours of establishing a metacognitive and discursive teaching and learning culture were decisive factors for these successes.

Based on the successes of MeDIM, new measures were developed and field-tested in the pilot study “Development and Testing of Teaching Methods and Material for the

Increase of a Long-Standing Success in Teaching at Secondary Schools in Indonesia” (NaMI), which were meant to enable Indonesian pupils of year 7 to sensibly use mathematical concepts and methods and to increasingly practise metacognitive and discursive activities.

NaMI is a design-based research and development project. This methodical approach is characterised by a theory-driven development of teaching and learning process and by empirical examinations of this development in order to contribute to theoretical understanding of learning in complex, real conditions. The developmental psychologist Brown (1992) shaped the term “Design-Based Research”, and this research method was widely spread in various teaching methodologies as “The Design-Based Research Collective” (2003). Papers of Cobb et al. (2003) are referred to as regards mathematical education.

The new “Learning Environments” for year 7 (new type of mathematical tasks and problems, didactic material and games) belong – among others – to the theory-based, empirically tested and validated measures in the project NaMI.

In this paper, several of these measures, the educational games, are presented. The selection of games was led by the idea that most of the presented games do not primarily require lessons following the principles of MeDIM or NaMI, but that they can be used independently of the textbook and the learning environments in Maths classes. To download the printing templates for the games (including Indonesian instructions) go to <http://www.ikm.uni-osnabrueck.de/reddot/460.htm>. In addition to that, there is a separate paper presenting a role playing game (RPG) for introducing the axioms of a group (Nowinska, 2013). This paper includes and discusses students' evaluations of the RPG.

To start with, we are going to describe the theoretical background used for the construction of the games explaining their intended effectiveness. Afterwards, the theory is used to present and interpret lesson parts or students' outcomes of the initial trials of the games. The lesson sequences and the students' outcomes are data which have been ascertained empirically within the project NaMI 2011 and 2012 in two classes 7, each in Solo (Jawa) and Weetebula (Sumba). A total number of 159 pupils of four project classes took part in this project.

### ***Theoretical Background***

Although the term (mathematical) game or rather play is commonly used both in everyday life and in academic literature, there is still a lack of a standard definition of this term. Some authors may see a great range of different meaning between games and plays and this paper does not differentiate between these two terms. When imagining the explained games the readers will have their own understanding of how we use this. However, we believe Leuders' (2009, p. 2) distinction between "Development Games" ("Erarbeitungsspiele") and "Practice Games" ("Übungsspiele") is useful for a fuller understanding of the intended effect of these games. "Development Games" are used to work out mathematical concepts to develop mathematical methods and reasoning and to discover relations between mathematical concepts. "Practice Games", however, are used to recall mathematical concepts and methods already known, to apply them in a new context and to reflect on them.

Regardless the choice of the term, there is a consensus in didactic literature upon the fact that the use of games in maths classes cannot only be valued as an entertainment factor, but also as a method to bring forward the understanding of mathematics. For more than 30 years empirical tests have been executed on that point (cf. Bright et al., 1985; Ainley, 1988; Holton et al., 2001). Pinter (2010, p. 74) sums up the influence of the use of games in maths classes as a result of her dissertation as follows: "Playing games is not only fun, but students can learn more effectively through activities and participation rather than passive instruction since they are usually better motivated and more active in reaching their goals. Games provide a visual representation of problems through manipulative operations in a social context. They can increase students' knowledge, and, in addition, they influence their cognitive and social development."

Although there are many different understandings of the term (mathematical) game/play, didactic literature agrees that the benefit of using games is to motivate the students and to promote their cognitive and social abilities for a better understanding of Mathematics. Therefore many cognitive, social, and motivational reasons are mentioned. To get a rough overview, have a look at (cf. Holton et al., 2001; Ainley, 1988). There is a global drive towards using the didactic potential of games as part of the learning and teaching process and examining their learning impact (Burdick et al., 2008; Holton et al., 2001). There are many downloadable games that can be used in

Maths classes, e.g. on the website <http://www.mathwire.com/games/games.html>. In European countries and also in Indonesia, games are used in Maths classes. Nasrullah & Zulkardi (2011) identify the advantages of using traditional Indonesian games for gaining mathematical abilities. However, unlike our games, some games focus solely on learning mathematical facts, e.g. “A Parrot Game” (cf. Zetra Hainul Putra et al., 2011), which promote the learning of number bonds to ten. It is our aim that through the games the students will develop an understanding of key mathematical concepts (e.g. the activity game “Jumping back and forth on the number line” or that recently named RPG). The games also motivate students to further develop and consolidate their (mathematical) knowledge (e.g. the competitive game) and to accept challenging exercises (e.g. the Dominoes or Triominoes).

The games developed by us are established supporting measures to further develop lessons and to improve pupils' skills. Their use in class is meant to train the pupils for activities and interactions which encourage the learning and understanding of mathematics, i.e. the enforced practice of metacognitive and discursive activities, the creation and sensible use of sustainable mental models for mathematical concepts and methods as well as the learner's positive attitude towards learning mathematics. Hereinafter the intended targets are explained in more detail. As regards a theory-driven argumentation and explanation of these activities for the enhancement of learners' mathematical competences we would refer to Kaune et al. (2011).

**1. Establishing a desired culture in classes with learning focused forms of interactions between students and stimulation of individual, cognitive activities of the learners during lessons**

The benefit of our games is that they stimulate efficient teaching and learning culture in Maths classes. By playing our games independent and critical thinking as well as performing metacognitive and discursive activities in classes are strengthened. Some of our games support social interaction through working in pairs or in groups.

**2. Creation of sustainable mental models for mathematical concepts and methods, introduction of realistic contexts and linkages to students' cognitions to enable their understanding of mathematical theories and their reasonable usage**

Some of our games (e.g. the RPG or the jumping game) are designed for learners to construct mental models for mathematical concepts and methods. In addition, these

mental models enable a (better) understanding of Mathematics (cf. Kaune et al., 2011, p. 18f.). The intended aim is that learners deal with abstract mathematical terms and difficult tasks in the same way they would do with familiar objects and meaningful activities.

Furthermore, learners are put into a realistic context, in which they are engaged in certain hypothetical roles, such as jumping along a board in the jumping game (Kaune et al., 2011, p. 27) or being a bank customer in the RPG managing a bank account (Nowinska, 2013). Working in realistic contexts arouses the intuitive knowledge of students, encourages them expressing hypotheses, suggestions, arguments, or complex reasons and questions. As a result, these activities contribute to a (deeper) understanding of Mathematics (cf. Perkins & Ungner, 1994, p. 3).

### **3. Facilitate individual, cognitive and metacognitive skills; support of constructivist learning process**

It is outlined in Kaune et al. (2011) why it is essential for the effectiveness of games that learners are cognitively engaged in activities; in this context, it is explained that learners have to deal precisely with external representations and activities on the one hand, and with (internal) perceptions and mental activities on the other hand which have been previously evoked by dealing with the external representations. Therefore, they have to be educated to be able to self-control the process of constructing, verifying and reflecting upon their knowledge. Some of the games are designed with the aim that learners realise the necessity and advantage of planning and control; and that they are motivated to organise and control their own thinking and the outcomes of it. A competitive context motivates students to plan their actions precisely (development of strategies) and to control their outcomes. These kinds of activities where students think about and control their thinking and have knowledge about their own knowledge are examples of what is called metacognition (cf. Flavell, 1976, p. 232). The category system for metacognitive activities in gradually controlled argumentation in maths lessons (Cohors-Fresenborg & Kaune, 2007) offers, as a result of many years of research, a tool to reveal and analyse metacognitive activities.

### **4. Challenging learning strategies motivate learners to apply and deepen their knowledge**

Our practice games (such as the Dominoes or the Triominoes) fulfil the didactic function of motivating learners to revise and deepen their mathematical knowledge

with hands-on activities. In using these activities where students are working with partners they are motivated to explain and understand each other's reasoning. Nevertheless, the students are not aware of that aim. Whilst this goal remains hidden for the learners, the materials surrounding the activities repeatedly lead to mathematical thinking and reasoning. And it is this mathematical questioning, which every student needs to answer to continue the activity successfully and the results of it. By providing students with the freedom to choose their own strategies they are more likely to demonstrate curiosity and to be intrinsically motivated. To complete these activities successfully, goal-directed actions in the partner or group work are inevitable. In addition, these goal-directed actions aim at student interactions (cf. item 1), which are otherwise very rare in Indonesian classes as one can see in the following quote by Sembiring et al. (2008, p. 929) on Indonesian Maths classes: “Students were expected to learn Mathematics in passive ways and, but some hardly learned it at all. Many students became used to being spoon-fed by their teachers, and were rarely asked to think creatively or critically about what they were learning.”

### **5. Change in the individual attitude towards Mathematics**

A pilot study with German pupils and observations of Indonesian pupils in our experimental classes suggest the hypothesis that pupils enjoy and are more engaged in Mathematics whilst playing games. Furthermore, their overall attitude towards Mathematics positively improves. To prove this, an independent study with Indonesian pupils intends to investigate this hypothesis further.

#### ***Introduction of Some Games: Concept and Anticipated Impact***

As mentioned above we follow the distinction between “Development Games” and “Practice Games” (cf. Leuders, 2009, p. 2). Below we will introduce the concept and expected effectiveness of three “Practice Games”. Furthermore, we will interpret and analyse some chosen lesson transcripts and students’ outcomes to document the effectiveness achieved.

#### ***Competitive Game***

On one hand, a competitive game serves active, diverse practice and the reflection on mathematical knowledge, and, on the other hand, it is especially well suited for the

establishment of requested rules and modes of behaviour of a cognitively activated teaching culture (Lipowsky et al., 2009).

Based on our experiences with Year 7 Indonesian Maths classes, the merit of competitive games is evident. Further possible applications for these games are mentioned with expected and subsequent effects explained. The idea of competitive games presumes that learners are separated into two (or more) groups, in which they preferably have to deal with the same task consisting of many independent subtasks. Although it is possible to use more than one task, this might cause problems with evaluating the students' performances if the level of difficulty of the tasks differs. However, tasks which are of similar difficulty but still can lead to different solutions leave ample scope for monitoring and reviewing activities. This is what is being illustrated in the following.

The number of subtasks equals the number of people within a group. Each learner focuses on one subtask. The game consists of two stages. Once the class has been divided up into smaller groups, each student gets his/her own subtask on which he/she has to work independently. Afterwards, the teacher presents all tasks separately (and one after the other) with the help of a projector. For each subtask, two students come to the front of the class (one student per group). They present their solutions and explain them. Now, both solutions are discussed and the other students evaluate these. The other classmates have to provide reasons for their decisions. Furthermore, differences in the strategies used while working on the task have to be explained and their appropriateness has to be rated. In addition, any existing mistakes are revealed and explained in order to improve the solutions in the end. It is essential that it is the learners of the two groups and not the teacher who carry out the described activities so that the desired effects can be achieved with this competitive game. In this stage of the game each student receives feedback about the appropriateness of his/her own solution; but at the same time, he/she has the chance to assess other solutions and to comment on them. Therefore, the requirements on the learners do not only consist of the goal of determining the correct solution of one part of the task, but also of active participation in all parts of the task and of a discursive and technical debate. For each part of the task that has been solved correctly the student wins a pre-defined number of points; on the other hand, the group of the student does not win any point if they present an incorrect solution. In the end, the group with the most points wins.

If one analyses the idea of the competitive game against the background of the self-determination theory, the following hypothesis can be formulated: the learners' cognitive analysis of the learning material is advanced through extrinsic factors, which is the wish to contribute with their own competences to the success of the whole group and, eventually, to gain a reward for the group. Empirical studies suggest that such cause of action can, in fact, hinder the intrinsic motivation and hence the quality of the learning process (Krapp & Ryan, 2002, p. 60). However, this criticism should be considered with caution in relation to the game presented below, as demonstrated by the following arguments.

Our competitive games were not designed for lessons in which the learners should gain new important knowledge; rather they were designed for these lessons in which the already acquired knowledge is used in individual work at first, and afterwards for reviewing the tasks solved by the classmates with the help of the whole group. Our experience shows that where learners are required to evaluate the solutions of the opposing group and to defend the suspicion of a mistake in their own group's solution using technical arguments result in remarkably high cognitive examination of the parts of the task. In addition, it leads to numerous metacognitive activities, very precise controls, many student interactions and to improving students' confidence in being accurate when challenged.

The merits of this method are its positive effects on the improvement of interaction quality within the class. This positive influence is evident from the metacognitive and cognitive student behaviours which are an important requirement for insightful learning within the theory of cognitive activation of learners (Lipowsky et al., 2009, p. 529). Our empirical results lead us to the assumption that this method can contribute to improve students' attitudes to practicing metacognitive and discursive activities. These activities represent the intended features of the teaching and learning culture that is to be continuously improved. To ensure effectiveness of these methods in the long term, further actions need to be taken.

The teacher's role is to skilfully choose the tasks at first, and to chair well-organised discussions to talk about the single parts of the task. Besides, the teacher would also be involved in challenging the solutions that are presented by the students.

It is important to add that a competitive game is not bound by a particular subject and its topic. In 2011, we used the method of a competitive game in Year 7 Maths classes

in Solo (Java). A task (cf. Figure 1) in the textbook, which we used during the project, provided a pattern for the task constructed for the game.

At this time, the students had already extended their intuitive knowledge about transactions of credit and debit on a bank account and understood the necessary mathematical concepts to enable them to work with integral numbers. Nevertheless, the notation of account transactions in the context of a mathematical formulaic language revealed that many students made syntax errors.

Complete the following table:

Account history	Short notation
I am Rp. 350.000 in credit and I withdraw Rp. 400.000.	$(350.000 - 400.000) = (-50.000)$
I am Rp. 350.000 in debit and I withdraw Rp. 400.000.	
I am Rp. 350.000 in debit and I pay in Rp. 400.000.	
I am Rp. 350.000 in credit and I pay in Rp. 400.000.	
I am Rp. 250.000 in debit and my bank credits my account with Rp. 300.000.	

Figure 1. Task 2.6 (excerpt) from Kaune & Cohors-Fresenborg (2011, p. 22)

Therefore, the tasks were formulated in different degrees of difficulty. Tasks with the least degree of difficulty were those that required a formalisation namely that new credit is paid in while already having credit on the bank account; on the other hand, tasks with the highest degree of difficulty were those that required a formalisation namely that debts were paid out from credit. In some parts of a task, just one correct answer was possible while there could be two correct answers in other parts as shown in Figure 2.

I am Rp. 250.000 in credit and I want to close that account.	
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Figure 2. Booking History With Two Possible Formalisations

The description of closing the account does not necessarily mean that 250.000 Rupiah have to be withdrawn. As an example, one could transfer a debt of 250.000 Rupiah to this account from another account. This leads to the two possible solutions  $(250.000 + (-250.000)) = 0$  and  $(250.000 - 250.000) = 0$ . Tasks of this nature motivate students to control and double-check the solutions of the other groups, because where results differ from your own solution, they arouse suspicions of a mistake. However, this is not always the case. Such processes

stimulate high cognitive analyses accompanied by the metacognitive activities monitoring and reflexion.

With the help of this task, the connection between bank transactions and the addition (deposit) or rather subtraction (disbursement) of positive numbers (balance in credit) and negative numbers (debts) should be strengthened on the one side. On the other hand, it encourages students to be syntactically accurate in their recording of mathematical computations as number sentences since just one single minor mistake (e.g. omitting a single bracket) results in a lower score.

Furthermore, moving from the realistic bank transactions to the mathematical notations and vice versa fosters the use of mental models as a tool for mathematical reasoning.

This task was played in groups where girls competed against boys. Since there were exactly 14 boys and 14 girls in a class, 14 subtasks were prepared. Each subtask was printed out twice and handed out on a strip of paper to one student in each group. The student was asked to record his solution on this strip of paper. While handing out the copies, we avoided handing out the same task to two students who sat directly next to each other.

In the second stage, both solutions for each subtask were shown to the class at the same time. All the other students were supposed to monitor the solutions and to control the translation process into the formulaic language. Figure 3 shows the two solutions of the eighth pair of students.

One can see that the first solution is syntactically correct while the second one has a syntax error, which is that the pair of brackets around 100.000 is wrong. Nevertheless, the evaluation of the semantic correctness shows different results. Here, it is the first version which has to be marked wrong since the formalisation does not fit the description of the transactions. However, the final result of the two solutions is correct and, therefore, we can assume that student number one calculated the new balance with knowledge of the banking world. That is why the problem did not arise during the calculation but in the process of translation.

Saya punya debit Rp. 130.000 dan saya memindahkan debit Rp. 100.000 dari rekening ini ke rekening lain	$((-130.000) + (-100.000)) = (-30.000)$
Saya punya debit Rp. 130.000 dan saya memindahkan debit Rp. 100.000 dari rekening ini ke rekening lain	$((-130.000) - (100.000)) = (-30.000)$

Figure 3. Formal Representation of Transactions Recorded by Two Students

"I am Rp. 130.000 in debit and I transfer Rp. 100.000 from this bank account to another."

The following transcript gives an insight into the effect of this game on teaching culture.

Jessica and Jodi go to the front of the class and present their answers regarding the following account history: I am Rp. 350.000 in debit and I withdraw Rp. 400.000. Jessica's solution is the first one, while the second one is Jodi's.

Saya punya debit Rp. 350.000 dan saya menarik Rp. 400.000.	$((-350.000) - 400.000) = 50.000$
Saya punya debit Rp. 350.000 dan saya menarik Rp. 400.000.	$((-350.000) - (-400.000)) = -750.000$

- 6  
8  
10  
12  
14  
16  
18  
20  
22  
24
- Teacher So, I (Miss Novi) did not want to say whether this task is wrong or right. You are the judges! Raise your hands when you want to comment on the solutions. Hands up! [...]
- Cindy The first solution is similar to number two, isn't it?
- Teacher Do you all agree?
- Student They are similar.
- Cindy I am, I am 350.000 in debts and I (...). Ahh yeah: this is a deposit.
- Silly me. [laughter]
- Teacher Okay, raise your hands!
- Jessica The first one is wrong.
- Rafael Jessica, this will will (...). What (...)?
- Teacher So the first one, Jessica?
- Rafael In Jodi's solution, minus 400.000 are subtracted; but in Jessica's only 400.000.
- Student Yeah, why is that supposed to be wrong?
- Teacher Which one is the correct one, or are all of them wrong?
- Student They are all wrong.
- Novi Hands up, Valent!
- Valent They are all wrong.

- 26 *Teacher*            *They are all wrong. What do you mean by that, Valent?*
- Valent*                *In my opinion, the minus before 400.000 is missing in Jessica's*  
28                        *version. In my opinion, (...).*
- Teacher*            *What is your answer now, so that Miss Linda can write that down?*
- 30 *Valent*                *Open bracket, open bracket, minus 350.000 (6 sec) are subtracted, eh*  
                              *bracket, closed bracket, are subtracted, open bracket minus 400.000.*
- 32 *Student*             *How can that be?*
- Valent*                *I don't know. (Mboah.) [dialect]*
- 34 *Peter*                 *400.000 are supposed to be subtracted.*
- Valent*                *I am confused.*
- 36 *Teacher*             *Try to repeat your answer once again.*
- Peter*                 *Open bracket, open bracket, minus 350.000 closed bracket, minus*  
38                        *400.000 are subtracted, closed bracket, equals 750.000, eh, open*  
                              *bracket, closed bracket, hehe, in there [means: inside the brackets]*  
40                        *minus 750.000 and also outside.*
- Teacher*             *Okay.*
- 42 *Student*             *Confused.*
- Teacher*             *So now who do you think is right?*
- 44 *Student*             *Nobody.*

In this short scene, which lasts only 3,5 minutes, at least seven students are involved. This discussion in class can now be interpreted from the point of view of various theories.

**The teacher's understanding of her role:** The teacher sees her role as being a host. Already in lines 5 – 7 it becomes obvious that she does not see herself as being solely responsible for the accuracy of the students' solutions. Instead, she repeatedly encourages the students to monitor the solutions independently (lines 6, 20, 41). Several times, she leads to a structured discussion (lines 6f., 13, 22) by asking the students not to shout into the class but to raise their hands instead. Even after the solutions have been reasoned on a factual level, she does not summarise what has been said but asks the students to do that (cf. line 41). Therefore, one can say that the teacher leaves a certain level of freedom leaving the learners motivated by the game environment and the familiar mental models which they can use to support their arguments. These comments on the role of the teacher are important in order to understand the positive effect of this game on students' learning activities, particularly on their active participation in the discourse and use of mental model as a tool for mathematical reasoning.

**Cognitive activation:** A positive correlation between the cognitive activation of learners in classes and the learning process was first proved in 2001 (Klieme et al., 2001). This achievement was the result of an analysis of the TIMSS video study. Many further studies of Maths classes have confirmed these results. Hence, the transcript should be considered within this theoretical perspective.

Cognitively activated lesson in the sense of Lipowsky et al. (2009, p. 529) can be described as follows: “In cognitively activating instruction, the teacher stimulates the students to disclose, explain, share, and compare their thoughts, concepts, and solution methods by presenting them with challenging tasks, cognitive conflicts, and differing ideas, positions, interpretations, and solutions. The likelihood of cognitive activation increases when the teacher calls students’ attention to connections between different concepts and ideas, when students reflect on their learning and the underlying ideas, and when the teacher links new content with prior knowledge. Conversely, the likelihood of cognitive activation decreases when (...) the teacher merely expects students to apply known procedures.”

In order to approximately understand the construction of cognitive activation, Lipowsky (2009, S. 93) provides different indicators. These indicators, on one hand, refer to how the teacher initiates and promotes this process of cognitive activation and, on the other hand, to how the learners use this offer and perform cognitively demanding operations.

In the transcript analysed there are noticeable characteristics of the cognitive activation of learners, namely students were comparing and assessing the solutions presented as well as the student interactions, as e.g. the request of one student to a further explanation in line 30. With this type of game, the cognitive activities were specifically demanded, but at the same time, the content of the subtask stimulated cognitive activities. We value them as characteristics of the intended development of the teaching culture. But since such developments are a long, complex and not linear process of improving the teaching and learning attitudes, we are not surprised by the not elaborated form of students’ activities and the missing of a precise explanation of Peter’s suggested solution. The learners use the mental models offered in the learning environment „bank” without inhibitions in order to explain mathematical notations and justify the results of their calculations.

**Metacognitive activities:** Even in her first contribution (line 8), there are suggestions that Cindy reflects upon her thinking: It may be that she compared both texts of the bank transactions (perhaps just focussing on the numbers) or maybe she just compared the structure of the number sentences to identify the similarities. In line 11, she begins on a factual level by determining the difference between the two transactions which she previously claimed to be similar. This can be described as a self-monitoring process. We interpret Cindy's comment "Ahh yeah: this is a deposit. Silly me" in lines 11f. as a monitoring activity.

The comments by Jessica (line 14), the student in line 21 and Valent (line 23, 25f.) show that the students were self-controlled and actively reflecting since they discovered the mistakes in the solutions. Compared to the first two students, Valent justifies his objection (line 25). Furthermore, he shows signs of self-monitoring (line 33). In line 17, we can see that Rafael reflected on the solutions since he compares both structures of the terms and highlights the differences between the two solutions. Moreover, the comment of the student in line 19 is remarkable. Rafael's comments do not seem to convince this student which is why he requires an explanation. Also, we interpret the student's question in line 30 as a request to the student (and not to the teacher) to expand on this explanation even more.

The analysis of this teaching sequence leads to the assumption that metacognitive activities of learners are initiated and supported by both the type and the content of the game with its realistic context. The rules of this game motivate the learners to control and to precisely analyse the presented solutions in order to gain as many points as possible for their own group. The control and the analysis succeed with the help of familiar mental models for mathematical operations on integers.

### ***Dominoes***

The Dominoes game is used in order to consolidate key mathematical concepts that have been previously taught in class, thus it is categorised as a practicing game. Nevertheless, it can only be used if the axioms of a group are known to the students and if the students have gained competences in calculating within the set of integral numbers ( $\mathbb{Z}$ ). Then, one can play the Domino individually but also in pairs.

**Game instructions:** All cards are laid out to the students so that their labels are legible. The goal of the game is to put the cards together such that the information on the right side of the (left) card matches

the information on the left side of the laid down (right) card. The first card to be played is the one which has got “start” printed on the left side.

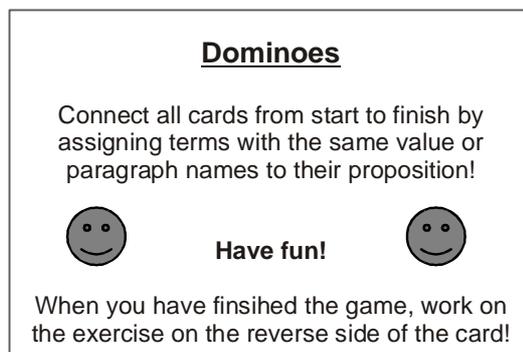


Figure 4. Game Instruction on the Dominoes

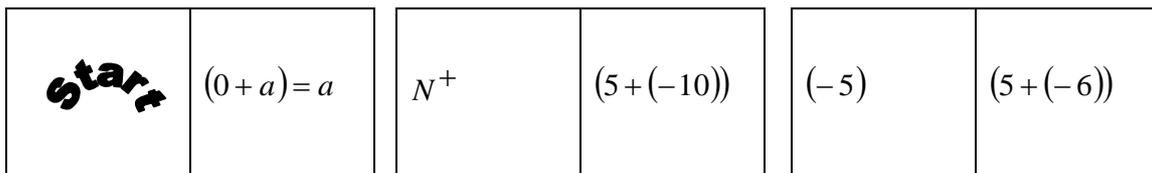


Figure 5. The First Three Dominoes<sup>1</sup> Laid Together

If the card with the game instructions is turned around, another task is revealed: “You have matched together the terms  $((b - b) + b)$  and  $b$ . Now explain step by step why each Domino is in the correct place”. This task is naturally differentiating as some students will finish the Dominoes very quickly in comparison to their classmates. The theorem  $((b - b) + b) = b$  is only provable in three steps if using axioms.

A competitive context can be introduced by setting a time limit or awarding this group of students which finishes first.

**Game material:** Our designed Domino game consists of 20 tiles. Each tile has a left and a right half labelled. The first and the last tile are indicated by the information “start” on the left side and “finish” on the right side. This information on the tiles differs in the following way:

- On four cards, abbreviations for the names of three group axioms interpreted for the addition and one definition are written on: the axiom of the identity element

<sup>1</sup> The abbreviation  $N^+$  stands for the name of the axiom “Identity Element of Addition”. The students have learned about this axiom’s form which is listed on the right half of the first Domino card.

$(\mathbb{N}^+)$ , the axiom of the inverse element ( $\Gamma^+$ ), the commutative property ( $\mathbb{K}^+$ ) as well as the definition of subtraction ( $\mathbb{D}^-$ ). Correspondingly, there are four cards labelled with the content of one axiom or rather one definition in mathematical terminology.

- 13 cards are labelled with terms on the left side consisting of only one number.
- 12 cards are labelled with terms on the right side that contain numbers and arithmetic operators.
- Two cards are labelled with terms on the left side that only consist of a variable: a and b.
- Two cards are labelled with terms on the right side that only consist of variable and arithmetic operators:  $(-(-a))$  and  $((b-b)+b)$ .

When using in classes, it is a good idea to use different coloured cardboard to avoid a mixing of cards from two or more sets being muddled (cf. Figure 6).

**Game strategies:** Before the students start the game they should look at the information on the tiles. Therefore, it's useful to advise the students to lay out all the cards in front of them so that they can have a look at them all at once. The teacher should ensure that the cards are not dealt in the correct order at the beginning of the game. The following photos show teachers playing Dominoes in partner work during a workshop in Cilacap Regency (Java). The space needed for playing Dominoes and possible arrangements of the tiles are clearly visible in the photos.



Figure 6. Teachers Playing the Domino in Partner Work During a Workshop in Cilacap Regency (Java)

In class, we were able to observe different game strategies:

1. At first, the cards with the phrase “start” or rather “goal” are sorted out and card after card is placed from the start card to the goal card. If one follows this strategy, the second step is to see whether the left half of the tile has  $N^+$  on it, the abbreviated name for the axiom of the identity element. The pairs of teachers in the photos above stuck to that strategy. A possible variation of that partner work could be that one student starts with the “start” tile and then proceeds “forward” while the other one starts with the “goal” tile and then proceeds “backwards”.
2. At first, the players are supposed to match pairs of tiles that belong together; afterwards, these pairs are stacked and eventually joined by moving to a complete chain from start to finish. Here, students can also take advantage of their own strengths in the first step by identifying those pairs which share information that can be easily matched. For example, they search for all paragraphs and the abbreviations of their names at first, or they search for all terms with numbers and arithmetic operators and the tiles on which the correct result is written on.

In figure 7, one can see Frido from Weetebula (Sumba) playing the game. He starts with the “start” tile and he has already matched the next three cards. In addition, one can see one pair laid aside on which the definition of the subtraction is written on.

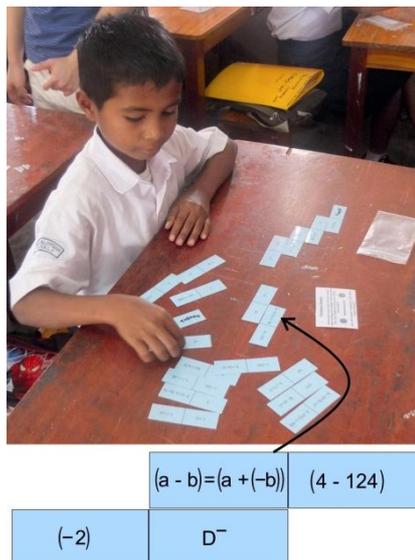


Figure 7. Frido Uses Strategies 1 and 2 While Playing the Game

**Analysis of classroom observation:** A possible objection to using this game might be that whilst the student is active he/she may not be cognitively active. This would be the case if they matched the cards randomly without taking time to consider whether they are appropriate matches. This has not been the case in our experiences. On the following left photo, Lusi's and Marta's (Wetebula, Sumba) facial expressions suggest that they are playing the Dominoes game seriously thinking about the next matching tile. The last card has already been put aside by the two players.



Figure 8. Lusi and Marta playing Dominoes seriously

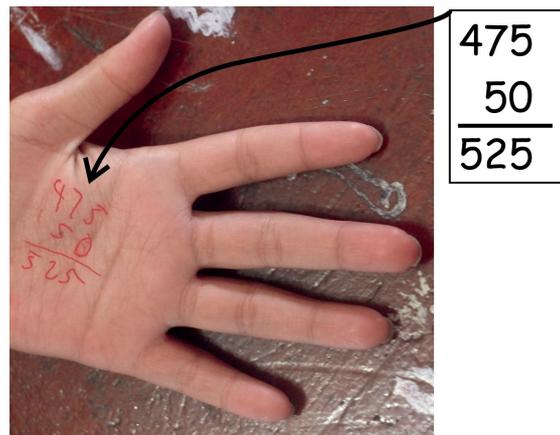


Figure 9. Chika Makes an Auxiliary Calculation on Her Hand

The students were often observed making auxiliary calculations. Figure 9 shows an auxiliary calculation that Chika from Weetebula (Sumba) made on her hand demonstrating how two tiles match together (cf. Figure 10). She thus determined the unity of the two tiles from Figure 10.

$(a + b) = (b + a)$	$((-475) + (-50))$	$(-525)$	$((-13) + 0)$
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Figure 10. Dominoes 13 and 14

On her auxiliary calculation, one can clearly see that she uses a strategy. She uses a mathematical theorem, which has not been discussed in class, which is that the sum of two negative numbers is determined by adding the amounts of the negative numbers and then forming the additive inverse of the sum of the amounts.

**Further considerations:** If one wants to increase the degree of difficulty, one can add further cards to the Dominoes, which can be appropriately attached to more than just

one card of the game. As a consequence, it would not be possible to apply strategy 2 to the same extent.

Our created Domino is designed to strengthen the usage of the agreement on adding and subtracting rational numbers only; but it is possible to develop a Dominoes for nearly any other mathematical topic.

### ***Triominoes***

Compared to the previously presented Dominoes, Triominoes have an increased degree of difficulty but still are of similar didactic advantage. Therefore, we developed three variations with different levels of difficulty, which differ both in the amount of cards as well as in the complexity of the terms on the cards. As a result and by using different Triominoes in Maths classes, it is possible to differentiate according to a student's internal needs. The Triominoes are also used as practice games, but they can only be used if the students know the axioms of a group. In this paper, we will only describe the Triominoes with a moderate level of difficulty. Triominoes can be played individually or in pairs.

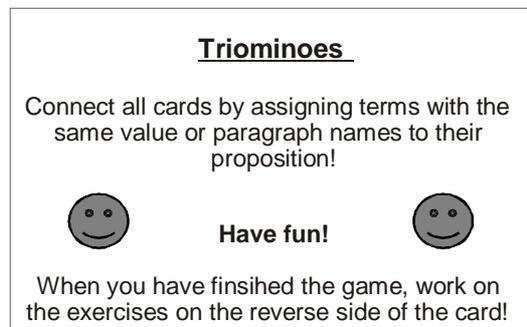


Figure 11. Game Instructions on the Triominoes

**Game instructions:** All cards are laid out to the students so that their labels are clearly legible. The goal here is still to put all the cards together in that way that the information on both sides of the two matched cards fits. The game ends when all the cards are placed together correctly. They then form a large equilateral triangle.

If the card with the game instructions is turned over, one can see additional tasks that can be used for the purposes of internal differentiation. Just like with the Dominoes, the students need to explain each match that they have made.

**Game material:** This Triominoes consists of 25 cards, which are not rectangles anymore but equilateral triangles. Some cards carry information on one side only,

some on two sides but the majority contains three kinds of information. The three cards, which only carry one piece of information, form the cornerstones; the nine cards with two pieces of information are on the edge of the triangle to be constructed. The following figure shows the three different types of cards in the Triominoes game.

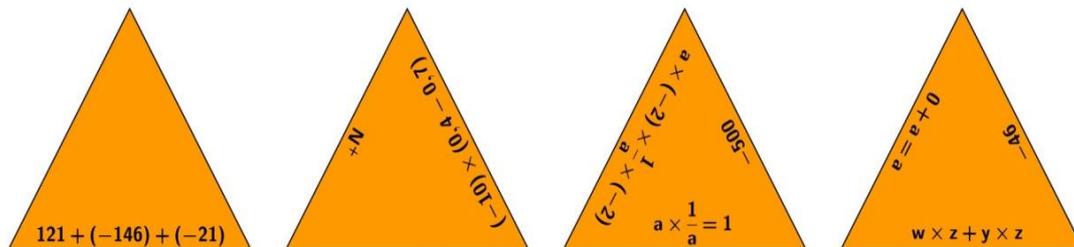


Figure 12. The Four Triominoes Cards Illustrate the Three Different Types of Cards on the One Hand, and the Different Pieces of Information on the Cards on the Other Hand

On the figure above one can see that the cards contain various types of information just like the information on the Dominoes:

- On seven cards, the abbreviations of the names of the group axioms are written on; one can see the abbreviation  $N^+$  for the axiom of the identity element of the addition on the second cards from the left.
- Corresponding to that, there are cards with the content of an axiom or rather a definition as a mathematical theorem written on them. The second card from the right is labelled inter alia with the axiom of the inverse element of the multiplication.
- Many cards are labelled with terms. The terms of varying complexity can be assigned to different groups: terms with only one number or a combination of number and arithmetic operator; terms with numbers, variables and arithmetic operators; and terms consisting of variables and arithmetic operators. There are two internal cards in the set which each contains term 3.



### **Summary and Prospects**

The empirical examples presented in this paper show that mathematical games encourage learners to practice metacognitive and discursive activities and to provide mathematical reasoning. These activities represent the intended features of the teaching and learning culture that is to be continuously improved. Empirical studies show that these activities enhance the mathematical performance of students. Furthermore, the examples provided in this paper show that students use the offered mental models for mathematical concepts and methods as a tool for mathematical reasoning.

Our intention is to examine metacognitive and discursive activities in Indonesian maths lessons in more detail and to investigate how playing games can contribute to improve students' attitudes towards these activities. Further research is needed to investigate how the mental models for mathematical concepts and methods are used by students as a tool to organize their mathematical knowledge and to make it understandable.

We are going to develop more games for Maths classes. A first game for recognising term structures and for practicing equivalence transformations of solving linear equations is already being tested. Furthermore, we are going to design more exercises based on mathematical educational theories, e.g. different types of number walls, which can be used to supplement the games in partner work stages.

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## Appendix

### Indonesian Transcript

Jessica dan Jodi datang ke depan dan meletakkan jawabannya sesuai dengan cerita pembukuan berikut: Saya punya debet Rp. 350.000 dan saya menarik Rp. 400.000.

Jawaban di atas dari Jessica dan jawaban di bawah dari Jodi.

Saya punya debet Rp. 350.000 dan saya menarik Rp. 400.000.	$(-350.000) - 400.000 = 50.000$
---	---------------------------------

Saya punya debet Rp. 350.000 dan saya menarik Rp. 400.000.	$((-350.000) - (-400.000)) = -750.000$
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- Guru: Coba, Bu Novi tidak ingin mengatakan benar atau salah. Kalian yang jadi jurinya. Tunjuk jari, yang mau jadi yang mau berkomentar. Tunjuk jari! [...]
- Cindy: Yang ini soalnya sama toh mbek nomer dua?
- Guru: Masak sih?
- Siswa: Sama.
- Cindy: Saya mempunyai saya mempunyai debet 350.000 dan saya (...). Ooo ini menyeter o'on. [Siswa ketawa.]
- Guru: Oke, coba tunjuk jari.
- Jessica: Yang atas salah.
- Rafael: Jessica itu di di (...). Apa (...)?
- Guru: Yang atas Jessica.
- Rafael: Yang Jodi, dikurangi minus 400.000, yang Jessica 400.000 thok.
- Siswa: Iya, kenapa harus salah?
- Guru: Yang benar yang mana atau salah semuanya?
- Siswa: Salah semuanya.
- Novi: Coba tunjuk jari. Valent!
- Valent: Salah semua.
- Guru: Salah semua. Menurut Valent?
- Valent: Menurutku, yang punya Jessica itu kurang min... 400.000. Menurutku (...).
- Guru: Jadi bagaimana, biar dituliskan bu Linda?
- Valent: Kurung buka, kurung buka, negatif 350.000 (6 sec) dikurangi eh kurung kurung tutup dikurangi kurung buka negatif 400.000.
- Siswa: Kok bisa?
- Valent: Mboh.
- Peter: Seharusnya dikurangi 400.000.
- Valent: Aku bingung.
- Guru: Coba diulangi lagi jawabannya!
- Peter: Kurung buka, kurung buka, negatif 350.000 kurung tutup dikurangi 400.000 kurung tutup sama dengan negatif 750.000 eh kurung buka kurung tutup hehe didalamnya negatif 750.000 dan diluar juga.
- Guru: Oke.
- Siswa: Bingung.
- Guru: Jadi siapa yang benar?
- Siswa: Tidak ada.