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DEVELOPMENT OF METACOGNITIVE AND DISCURSIVE ACTIVITIES IN INDONESIAN MATHS TEACHING

A theory based analysis of communication processes

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Abstract

We report on a German-Indonesian feasibility study which aims to significantly increase the mathematical skills of Indonesian secondary school students. For this study a learning environment for basic secondary school mathematics in class seven has been developed. It focuses on fostering cognitive, metacognitive and discursive activities. For the effectiveness of the new instructional concept it is necessary that those activities are an important feature of the teaching and learning culture in the classroom instruction. In this paper we present the theoretical framework for the new approach to teaching and learning. We use two transcript-based examples to exemplify and explain the observable features of this classroom culture und to formulate consequences for the following instruction development.

Keywords: classroom culture, metacognition, discursivity, cognitive activation

I. INTRODUCTION

For many years Indonesian mathematics educators have been striving for a reform of mathematics instruction, in order to improve mathematical skills of the Indonesian students. The most important and well established project in the reform process in primary schools (classes 1 to 6) is PMRI (Sembiring et al., 2010). In a German-Indonesian feasibility study („Development of metacognitive and discursive activities in Indonesian Mathematics" (MeDIM)) for the improvement the mathematical skills of pupils of class 7 and older cognitive, metacognitive and discursive activities should be fostered and thereby also the proven ideas from PMRI should be expanded. The theoretical framework of MeDIM, the conception of the new learning environments and tasks, and also the first study results are documented (Kaune, 2012a, b).

In the feasibility study two learning environments for the introduction of integers were designed. Both of them use a realistic context. We justify their learning effectiveness by the way, how the realistic context is used for the construction and organization of the mathematical knowledge in pupils' minds and for the introduction to the handling of mathematical theories. In the first learning environment we created a banking environment which is mainly based on balancing debt and credits. In the second learning environment we have modified the game “Hin und Her” (“back and forth”), a jumping game introduced into the German didactics of mathematics.

In Kaune et al. (2012a) it is explained why it is necessary for the effectiveness of the learning environment that pupils are cognitively, metacognitively and discursively active in classroom discussions. They should think deeply about external representations and activities in a realistic context on the one hand, and about their own mental activities and internal representations constructed on the basis of those external representations and activities carried out in the realistic context on the other hand. So, they have to regulate and control the process of their own knowledge construction and to reflect on this process. Activities of this kind, i.e. thinking about (one’s own) thinking, the regulation of (one’s own) thinking and also the knowledge about (one’s own) knowledge are called metacognition (cf. Flavell, 1976, p. 232). Our new concept of teaching and learning mathematics includes mathematical tasks which evoke students’ metacognitive activities and support cooperative and discursive working ways (Kaune et al., 2012a). Since the construction of new materials and tasks is not sufficient to improve students’ mathematical skills (cf. Sembiring et al., 2008, p. 928), there is also a need to establish metacognitive and discursive teacher’s and students’ behavior as a socio-mathematical norm in communication processes in the classroom.

The results of a qualitative and quantitative analysis of pupils’ learning effects from this feasibility study (Kaune et al., 2012b) show that students who participated in the project MeDIM achieved qualitatively better results in mathematical arguing and in handling whole numbers than the students of the control group.

For a better understanding of the efficiency of our instructional concept it is important to get more detailed information about the implementation in school practice, in particular about the practice of cognitive, metacognitive and discursive activities in the classroom instruction. On the basis of such an analysis it can be assessed if the implementation of the constructed learning environments was as intended. This evaluation results are important for following implementations.

An instructional development, in particular a change in teaching and learning mathematics, “a new way of thinking about the purpose and practices of school mathematics” (Sembiring et al., 2008, p. 928), “a new role of the teacher and new social and socio-mathematical norms" (Sembiring et al., 2008, p. 939) are recognized also by the Indonesian mathematicians as a change for a sustainable improvement of students' mathematical skills. This idea and innovative approach in mathematics education are in line with international discussion and research results about a correlation between instructional features and students learning outcomes. In this discussion, the role of metacognition (Wang, Haertel & Walberg, 1993) and cognitive activation (Lipowsky, 2009) is even more frequently addressed.

Metacognition und discursivity

Since Pólya (1945) a learners' activities in solving mathematical problems have been analysed. From this, the construct of metacognition evolved in the field of cognitive psychology. Our decomposition of the concept "metacognition" is based on these ideas. It is precisely described in a category system to classify stepwise controllable reasoning (Cohors-Fresenborg & Kaune, 2007).

An important component of metacognition is seen in planning problem-solving steps, including the choice of suitable mathematical tools. In addition, in the process of problem-solving the application of these tools has to be controlled, subject relevance and target reference have to be monitored and what is already achieved has to be compared to the target in mind. This activity of control and surveillance is named "monitoring" which distinguishes from a mental activity (called "reflection") that concentrates on the understanding of a given problem or on the reflection of intermediate results.

As we have extended the focus from problem solving to concept formation and understanding as well as from an individual perspective to interactions in class, additional mental activities have to be considered, which we subsume under the category of reflection: Reflection on the adequacy of concepts and metaphors, on the choice of the mathematical approach, on conceptions and misconceptions, and on the interplay between what was said, meant, and intended (the presentation and the conception). The control of arguments has been added to the category "monitoring". In the category "planning", planning metacognitive activities plays a role, as it might occur for example by choosing a suitable task or presenting a student's solution at the beginning of a new instruction section.

A deeper understanding of concepts, procedures chosen and applied tools is only possible if the monitoring and reflection precisely refer to what is discussed in class at the moment. A contribution's reference point has to be made obvious to those involved in the lesson's discourse and the understanding of what is said has to be supported by an adequate choice of words. We subsumed the activities essential for this under the notion of discursive activities. A discursive teaching and learning culture plays a crucial role in encouraging metacognitive activities of learners, especially in classroom communication, also in a social context that, from a constructivist point of view, influences the individual and socially shared learning processes.

In order to read and write mathematical knowledge accurately and to reason the ability to realise and articulate the difference between what has been presented and what had been the intension in the mathematics lesson is essential. For this purpose it is necessary to follow the lines of an argument, estimate its applicability and to strategically place doubt and counterarguments. This shows that metacognitive and discursive activities have to be interwoven.

Research has shown that metacognitive and discursive activities play an important role as subject independent indicators for the teaching quality. An overview can be found in Schneider and Artelt (2010). Wang, Haertel and Walberg (1993, pp. 272f.) emphasise the relevance of metacognition for learning achievements in general. In their meta-analysis of empirical studies on the success of school learning, they observe that metacognition should be listed on a high rank regarding the influence on learning achievements.

Cognitive activation

From a constructivist point of view, teaching can be successful - in terms of supporting students' deeper understanding - if the instructional methods involve students' cognitive activity (Mayer, 2004) and promote students' deep thoughts about the learning subject. Those instructional features can be described with the term „cognitive activation":

> "In cognitively activating instruction, the teacher stimulates the students to disclose, explain, share, and compare their thoughts, concepts, and solution methods by presenting them with challenging tasks, cognitive conflicts, and differing ideas, positions, interpretations, and solutions. The likelihood of cognitive activation increases when the teacher calls students' attention to connections between different concepts and ideas, when students reflect on their learning and the underlying ideas, and when the teacher links new content with prior

knowledge. Conversely, the likelihood of cognitive activation decreases when (...) the teacher merely expects students to apply known procedures” (Lipowsky, 2009, p. 529).

Cognitive activation does not only mean behavioral activities but mental activities aimed at the understanding of learning content, mathematical terms, methods und results. The efficiency of „cognitive activation” is theoretical founded and explained (cf. Lipowsky, 2009, p. 94), thus it can be expected that it will provide positive learning results.

II. RESEARCH METHOD

We investigate the effectiveness of the constructed instructional concept by using qualitative and quantitative methods in analyzing students’ learning results on the basis of their written works (Kaune et al., 2012b) and, additionally, we conduct an investigation of the implementation of the concept into the school practice and thus of the instruction development. Our research interest is to understand positive and negative factors that influence the implementation and thus the effectiveness of the learning environments. Because of this interest we conduct a qualitative, theory based, investigation of classroom communication. Results from this investigation provide an explanation for the achieved learning results. This investigation plays a crucial role for the following implementations of the instructional concept.

III. RESULT

In the following two instruction scenes from the feasibility study will be theory-based analyzed.

Scene 1: In class the students have had to simplify the term ((-3)-(-2)). One of the students proposes the solution -5. In order to understand the following discussion it is necessary to know that at this time the notation -5 had been meaningless, according to the syntax rules one had to use parentheses (-5).

1 Dyah	The next one is.. is wrong.. it should be with brackets… That is, that minus three, that minus three… is subtracted by minus two. The minus becomes plus. Therefore minus two is added to minus three is equal to minus five. But, in my opinion… the answer is correct, but in mathematics we need brackets. So, it is wrong..
5 Teacher	Okay, who wants to give opinion?
6 Some students	That’s wrong, miss...
7 Other student	I have the same (opinion), I have the same (opinion) like Dyah
8 Teacher	Okay, Dodi... Please, Dodi...
9 Ruben	Need brackets.
10 Dodi	Me, me.. Please, me..

11 Teacher	Yes.
12 Dodi	First we look to that side, miss, to left, correct miss?
13 Teacher	Yes.
14 Dodi	Because we got minus, we look to the left, then we got minus, we have to walk backwards, miss…
16 Teacher	Yes.
17 Dodi	Two steps backwards. Three minus two is one. Thus, minus one.
18 Siswa	Minus one.
19 Teacher	Dodi said, the result is minus one.
20 Some students	Correct.

Starting point of the brief discussion is an incorrect student solution. This solution is analysed by several students and is corrected in two steps via an intermediate result, without any factual information or an evaluation of the result provided by the teacher. The teacher understands it as her role to encourage the students to tackle the erroneous solution. The scene shows how the students in the project class deal with formal representations and that for their explanations they make use of a model world in which they are able to construct the meaning of the formally presented objects.

With respect to **metacognitive activities**, one can observe that Dyah (l.1-5) first carries out a syntax check and thus monitors the notation of the solution, and then, in a second step, checks the calculation. Even though she does not reach the correct solution at the end of the monitoring activity, it must be noted that she exhibits different facets of monitoring activities. In line 8 several students show their agreement with her judgement; they as well must have engaged previously in checking and monitoring activities. Other students agree with Dyah as well, referring to the reference person. Embedding contributions in the context, for example by mentioning the reference person as it occurred in this discussion, are means to facilitate understanding in the class discourse.

Dody's query to the teacher in line 14 has to be regarded as a discursive activity. We interpret it as an affirmation, a request to confirm what he had just said.

Possibilities for improvement: Linguistic inaccuracies in student utterances are not beneficial to the classmates' process of understanding: "The minus becomes plus" is an insufficient description of $((-3) - (-2)) = ((-3) + 2)$. Dyah does not use the definite article appropriately as its use requires the object "minus" to be unambiguously identifiable. Dodi's explanations in lines 12, 14f. and 17 are as well incomplete in their argumentation. Instead of "First we look to that side, miss, to left…" (l.12), to help his classmates to follow his argument, he should have first established a connection between the formal representation and his association in the model world of the game of dice. Then his argumentation in line 14 could begin as follows: *We find ourselves on the field (-3). The sign dice shows "minus". Therefore, we turn around to look to the left. Since the number dice shows a negative number we have to go backward.*

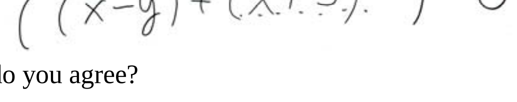
His utterance in line 17 is difficult to understand: In the first sentence he refers to the jumping game. In the second he talks about the term (3 - 2) instead of ((-3) - (-2)). One could interpret this in such a way that he is, in his model, still in the range of negative numbers on the board. But he does not express this verbally. From his third sentence one cannot infer whether "minus one" is meant to refer to the result or the field -1 in the game. Only the teacher points out in line 20 that she understood -1 to be a result of the calculation.

To use the mistakes in a constructive way, it would have been desirable if the teacher at the end of the conversation gave an impulse to analyse with the students on a meta level what might have let to the first result -5. This would have been a chance to encourage further metacognitive activities, especially reflection. Reflection is needed to explain and correct the mistake of Dyah. We suppose that the student has a false rule for subtraction by a negative number in his mind. This false rule was not discussed in the communication process and, thus not corrected.

Scene 2: In the focus of the following scene from a lesson is the mathematical action "term substitution". The ability to substitute a term for a variable and to realise and explain the effect of such a process is required for example for the sensible use of mathematical formulas (theorems). In class the axiom to form the inverse has been introduced in the model world "Debiting and crediting" as the paragraph I^{+} ($a + (-a)) = 0$, used to close a bank account. The term (x - y) has to be substituted for the variable a.

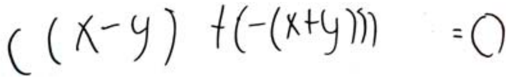
The scene shows, that the students in the project lessons confront formal representations without fear, that they themself express the demand for further explanations and by this show independent thinking and their commitment to comprehension. The little elaborative use of language does not allow for a detailed analysis of the student and teacher utterances or thought processes.

1 Dyah	*[Writes her solution on the whiteboard: ((x-y) + (x+y)) = 0]*

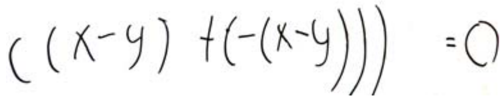


3 Teacher	Okay, do you agree?

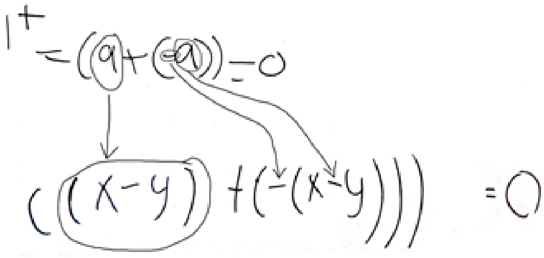
4 Student	No.
5 Teacher	No? Is there e... any other opinion? (6sec) Dodi? Come on, why is Dodi always shy? (7sec) Anybody with another opinion? (20sec) Vendi? It is okay, just try. (12 sec) Viola? Come to the front, Viola? *[Viola refused]* Why? Okay, first I want to question you: Is there anybody here who agrees?
9 Student	No.
10 Teacher	All of you do not agree, but why don't you give your opinion?
11 Ruben	We have done it (on paper).
12 Teacher	What did you say, Ben? Come here, Ben... here, Ben...
13 Ruben	*[Ruben comes to the front and writes his solution: ((x-y) + (- (x+y))) = 0] (32 sec)*



15 Teacher	Do you agree? (6 sec) Different solution, Ndo? What is the difference, Ndo? (27 sec) Ndre!
	[Andre comes back to the front of the class and makes a correction]
18 Andre	Why did he change this into plus? *[points to Ruben's equation]*
19 Teacher	How should it be?
20 Andre	*[Writes a = (x - y), then erases the plus sign in the term - (x + y) and three closed brackets in Ruben's equation, writes minus in place of the erased plus sing and rewrites three close brackets. Gives the pen to the teacher and goes back to his seat]*.



24Teacher	Can you explain, Ndre?
	[…]
34 Andre	Friends, we can see here, *a* is: in a bracket *x* minus *y*. We use the formula *I*, the agreement *I*. Correct? So, this *a [points to the first a in the equation (a + (-a)) = 0]*, this *a* replaces this *x* minus *y*. This is another *a*. A minus, a min. This *a*, it does not have its own brackets and this one - which replaces the vari… variable - has its own brackets, this minus sign is separated. So, actually it replaces *x* minus *y*. This minus is separated. In separated room. Equals zero. *[points to the equations (a + (-a)) = 0 and ((x - y) + (-(x - y))) = 0, draws something on the whiteboard, but his drawing is seen in video first at the end of his explanation.]*



Starting point of the class discourse is Dyah's erroneous solution (l. 2). Several students judge the solution as false without providing a reason, thereafter Ruben makes some changes but does not yet correct it properly. Only the student Andre finds and corrects the mistake (ll. 20ff). The teacher does not judge Andre's solution. She asks the student to provide an explanation (l. 24). Accordingly Andre explains his approach to the term substitution. His long explanation is imprecise but well-structured. The following excerpt of the discussion is not shown in the transcript: Since the student Panta expresses her lack of understanding, her classmate Nadia explains the term substitution again in her own words. At the end of this conversation a remarkably direct student-student interaction arises, which shows an intellectual engagement with the matter discussed: Nadia asks her class mate, whether she understood her answer:

Nadia	Friends... *[other students are laughing]*. This *a* is replaced with this *[unseen in the video]*. Then, this *a* also with this. Then this, this minus we move to here, this *a* to here. The result is zero. *[Asks Panta:]* **Clear?**
Panta	Yes.

An analysis of this scene with regard to **cognitive engagement** allows to formulate characteristics of a cognitively activating teaching culture. The teacher considers it has her role in the class discourse to obtain and record student contributions (false solutions as well) as a basis for further discussion (ll. 5-8, l. 15), to relate student solutions to each other and to encourage students to explain differences in their solutions (l. 15). The teacher does not voice whether the examination of the student solutions is correct.. By requesting the examination of the present solutions (l. 8, l. 15) and reasons and explanations (l. 10, l. 24) the teacher emphasises that she wants to teach the students independent and critical thinking.   She shows furthermore that she expects such a behaviour from her students. (“All of you do not agree, but why don’t you give your opinion?”, l. 10).

The students respond to contributions from their fellow students (l. 18). But one can see the learners’ difficulties to provide reasons for their solutions and to formulate their statements in a precise way.

The teacher’s effort to use the student solutions in the class discourse in a constructive way and the fact that the students address the existing solutions are evidence for the process of a positive class development. In the next step of this development thought processes and models, on which the student solutions are based, should be articulated more clearly, assessed and if necessary readjusted. This is necessary to explain, understand, and ultimately correct and avoid errors and misconceptions. Dyah’s and Ruben’s erroneous solutions have been corrected in this scene, but possible reasons have not been determined. Thus, one cannot reliably establish that both the students have understood their mistakes or false thinking strategies.

An analysis of the scene with respect to **metacognitive and discursive activities** reveals characteristics of the desired way of class culture. The teacher motivates her students to monitor. The students engage in monitoring by checking the present contributions of their classmates and voice comprehension difficulties (l. 18).

Later in the class conversation the teacher asks the students to critically asses themselves. They should examine whether they understood their classmates explanations („*Okay, is that clear? Who got helped from Andre's explanation?*“). So the teacher does encourage reflection – encourages thinking about one's own thoughts, encourages to evaluate the own comprehension process. Some students, as for example Panta, show that they can express their lack of understanding without having to be ashamed („*I do not understand.“).*

Discursive activities are only seen in form of naming reference persons. With the help of further discursive activities, unfortunately missing here, one could have structured the conversation in a more comprehensible way and the cognitive potential of student solutions and verbal contributions could have been utilised even better for the learning process. First of all we have the following activities in mind: more precise naming of reference points (e.g in Andre's contribution (l. 34)), highlighting the differences in solutions and solution strategies.

IV. DISCUSSION

The analyzed scenes stand out clearly against the picture of a ‘typical’ Indonesian mathematics instruction in which „the students give the answers in chorus. The teacher makes a response by saying ‘good’ whenever the students come up with the right answers, but he does not comment if the responses are wrong“ (Sembiring at al., 2009, p. 929). In both scenes the teacher show her ability to guide the learning process of the students more indirectly than directly, that is to manage the learning activities by designing interactive learning environments (cf. Reusser 1995, p. 23). In this context, the gradual handing over of control is a crucial effort that a teacher has to provide to construct an interactive teaching-learning environment. In this sense the teacher is no longer predominantly a mediator of knowledge, but an expert in matters of thorough understanding. As Reusser pointed out, this only looked like a control loss, at first sight, on closer glance, however, it is to be assessed as the more challenging task.

The students' behaviour shows that they are not used to provide reasons for their solutions without being asked for and to present their thought processes as precise as possible in the communication process, thus making them available to be monitored by their classmates. However, since we are dealing with a development process of a teaching and learning culture, we derive from our critical remarks further recommendations for the development of positive students' and teacher's behavior: Students should respond more frequently to other students, their possible ways of thinking, conceptions and misconceptions, control these and also reflect on their own conceptions to mathematical contents.

Our statement is that the constructed learning environments support this kind of activities, because students first build an evidence basis for mathematical knowledge on the basis of their experience in a realistic context. This familiar knowledge basis (for example about booking debts and assets with a bank) motivates the students to argue and to communicate intuitively understood knowledge. Those activities can even be practiced when in the class discussion contents from abstract mathematics are brought up for discussion, because, on the one hand, the necessary intellectual behaviour has been practiced successfully on a familiar knowledge basis and, on the other hand, there exists an evidence basis for mathematical contents, to which a pupil can fall back again and again, in order to describe these mathematical contents in an understandable way. When falling back to the evidence basis, metacognition must be practiced.

For a better understanding of how metacognitive and discursive activities in Indonesian mathematics teaching practices can be strengthened, more detailed analysis of communication processes in the classroom are necessary. This will be continued in a successor study.

V. CONCLUSION AND SUGGESTION

On the basis of a theory based instruction analysis learning-enhancing features of class discussion in mathematics education in Indonesian grade 7 could be explained and consequences for further instruction development could be formulated. Since the learning effectiveness of the designed learning environments is dependent also on the way of their implementation in instruction practice, we assess the already obtained students' understanding of mathematical contents and the qualitative differences between the project group and the control group as a success. These learning results allow us to expect that even greater learning results can be achieved if the formulated consequences for the instruction development are converted.

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